This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : 6714

Unique Paper Code : 32371302 HC

Name of the Paper : Survey Sampling and Indian Official Statistics

Name of the Course : B.Sc. (H) Statistics

Semester : III

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all,

selecting *three* from each Section.

Section I

1. (*a*) Define simple random sampling without replacement from a finite population. Derive the unbiased estimator of the population mean and find its sampling variance.

(*b*) For srswor, prove that :

$$\text{cov}(x_i, \bar{y}_n) = \frac{N-n}{Nn} \cdot \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X}_N)$$

$$(Y_i - \bar{Y}_N) = \frac{N-n}{n(N-1)} \text{cov}(X, Y)$$

Also evaluate $E(\bar{x}_n, \bar{y}_n)$. 6,6½

P.T.O.

2. (*a*) Compare regression estimator with ratio estimator and simple random sample mean, assuming the formulae for the variances of the estimators.

(*b*) Prove that the mean of cluster means $\bar{\bar{y}}$ is an unbiased estimator of population mean with variance given as :

$$V(\bar{\bar{y}}) = \frac{N-n}{N-1} \cdot \frac{\sigma^2}{nM} [1 + (M-1)\rho]. \qquad 5,7\frac{1}{2}$$

3. (*a*) Obtain the estimated relative efficiency of cluster sampling with respect to srswor.

(*b*) Define difference estimator and derive from it the regression estimator. Also obtain the variance of regression estimator under first approximation. $5,7\frac{1}{2}$

4. (*a*) If y and x are unbiased estimators of the population totals of Y and X respectively, show that the variance of ratio estimate $\frac{y}{x}$ can be approximated by $C_y^2 - C_x^2$, where c_x and c_y are coefficient of variation of x and y respectively. (The correlation coefficient between $\frac{y}{x}$ and x is assumed to be negligible).

(*b*) From a simple random sample of size n drawn from N units by srswor, a simple random sub-sample of n_1 units is duplicated and added to the original sample. Show that the mean based on $(n + n_1)$ units is an unbiased estimator of the population mean. Also obtain its variance. How does it compare with the variance of the estimator based on n units only. $6,6\frac{1}{2}$

Section II

5. (*a*) Derive the variance of the estimate of the population mean based on systematic sampling in terms of intra-class correlation coefficient ρ. Prove that reduction in this variance over srswor will be 100% if ρ assumes the minimum possible value. If ρ assumes the maximum value, what is the relative efficiency of systematic sampling over simple random sampling ?

(*b*) Justify the following statements :

(*i*) The smaller the size of stratum, the smaller should be the size of sample to be selected therefrom.

(*ii*) The smaller the variability within a stratum, the smaller should be the size of sample selected from the stratum.

(*iii*) The cheaper the cost per unit in a stratum, the larger should be the size of sample selected from that stratum.

Hence obtain minimum size required for estimating population mean with fixed variance under optimum allocation. 6½,6

6. (*a*) Discuss briefly the present statistical system in India.

(*b*) Write about National Statistical Commission in India mentioning its *two* important functions.

(*c*) Name *two* Government of India's principal publications each on population and industry. 5½,4,3

7. (*a*) Obtain the estimated gain in precision due to arbitrary stratification over simple random sampling without replacement.

(*b*) Write short notes on the following :

(*i*) The States' Statistical systems

(*ii*) Economic Census

(*iii*) Objectives of NSSO. 6½,6

8. (*a*) With two strata, a surveyor would like to have $n_1 = n_2$ for administrative convenience instead of using the values given by Neyman's allocation. If $V(\bar{y}_{st})$ and $V(\bar{y}_{st})_{opt}$ denote the variances of the estimate of population mean under stratified sampling with the condition $n_1 = n_2$ and under Neyman's allocation respectively, then show that the fractional increase in the variance is :

$$\frac{V(\bar{y}_{st}) - V(\bar{y}_{st})_{opt}}{V(\bar{y}_{st})_{opt}} = \left(\frac{r-1}{r+1}\right)^2$$

where $r = n_{1(opt)}/n_{2(opt)}$ and f.p.c. are ignored.

(*b*) Define linear systematic and circular systematic sampling. Prove that systematic sampling is more precise than srswor if the variation within the systematic samples is larger than population variation as a whole. 6½,6